Mathematics

Class X

Chapter 7 : Theorems Related To Angles In A Circle

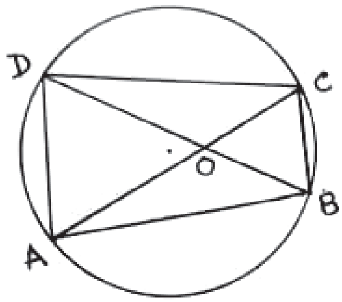
1. If in a cyclic quadrilateral ABCD, AB = DC then prove that AC = BD.

Ans. Given : In the cyclic quadrilateral ABCD, AB = DC and the two diagonals AC and BD are intersected at the point O.

R.T.P. : AC = BD

Proof : $\angle BAC = \angle BDC$ [angles in the same segment]

$\therefore$ $\angle BAO = \angle ODC$



In the ΔAOB and ΔODC, $\angle BOA = \angle COD$ [vertically opposite angles]

$\angle BAO = \angle ODC$ [Proved]

AB = DC

$\Delta AOB \cong \Delta ODC$ [AAS]

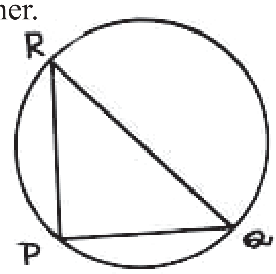
$\therefore$ AO = DO and BO = CO [Corresponding sides]

$\therefore$ AO + CO = DO + BO

$\therefore$ AC = BD.

2. Two chords PQ and PR of a circle are mutually perpendicular to each other. If the length of the redius of the circle is r cm., then find the length the chord QR.

Ans. In the circle, PQ and PR, the two chords are mutually perpendicular to each other.



$\therefore$ $PR \perp PQ$

$\therefore$ $\angle QPR$ is the angle of semicircle

$\therefore$ QR is a diameter of the circle

$\because$ Radius of the circle is r cm.

$\therefore$ QR = diameter = 2r cm.

3. O is the circumcentre of isosceles triangle ABC and $\angle ABC = 120^o$; if the length of the radius of the circle is 5 cm., then find the value of the length of the side AB.

Ans. O is the circumcentre of isosceles triangle ABC and $\angle ABC = 120^o$.

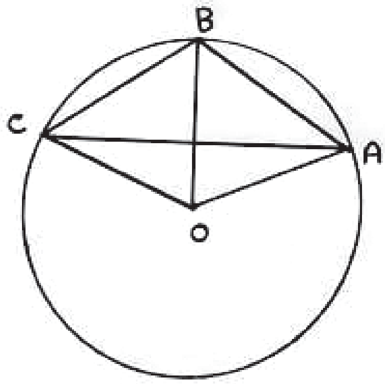
Join A, O and C, O

Here AB = BC

In the ΔAOB and ΔBOC.

OA = OC [radii of the same circle]

OB is their common sides and AB = BC



$\therefore$ $\Delta AOB \cong \Delta BOC$ [SSS]

$\therefore$ $\angle ABO = \angle CBO$ [corresponding angles]

$\therefore$ $\angle ABO = \frac{120^o}{2} = 60^o = \angle OAB$ [radii of the same circle]

$\therefore$ ΔOAB is an equilateral triangle.

$\therefore$ AB = 5 cm. [$\because$ radius = 5 cm.]